17/4/2020 (3)



## Union of Closed Sets

M.9

Th :- If $F_1$ and $F_2$ be two closed subsets of a topological space $X$. Then $F_1 \cup F_2$ is a closed set

Proof :- $F_1, F_2$ are closed sets.

$\Rightarrow F_1', F_2'$ are open

$\Rightarrow F_1' \cap F_2'$ is open

$\Rightarrow (F_1 \cup F_2)'$ is open (De-Morgan Law)

$\Rightarrow F_1 \cup F_2$ is closed.

$F_1, F_2, \ldots, F_n$ are finite No of closed subsets of $X$, then their union will be a closed subset of $X$.

Ex. :- an infinite collection of closed sets in a topological space is not neccessarily closed.

Sol: Let $(R, U)$ be usual topological space and let $F_n = [\frac{1}{n}, 1]$, $n \in N$, $F_n$ is a closed interval of $R$. Now

$$\cup\{F_n : n \in N\} = \{1\} \cup \{\tfrac{1}{2}, 1\} \cup \{\tfrac{1}{3}, 1\} \ldots = ]0, 1]$$

as $N = \{1, 2, \ldots\} \rightarrow$ Natural No's.

$]0,1]$ is not closed

Here we applied $\lim_{n \to \infty} \frac{1}{n} \Rightarrow 0$ as $n \to \infty$

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M.S.

**Neighbourhood :-** Let $(X, T)$ be a topological space and $x \in X$. A subset $N$ of $X$ is said to be T-nbd of $x$ iff $\exists$ T-open set $G$ such that
$$x \in G \subseteq N$$

Ex :- Let $X = \{1,2,3,4,5\}$
$$T = \{\phi, \{1\}, \{1,2\}, \{1,2,5\}, \{1,3,4\}, \{1,2,3,4\}, X\}$$

Here we are going to find T-nbd. of '1'.

~~subsets of X containing '1' are~~ nbd's of '1' are

$\{1\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}$ and $X$.

Now in above subsets of X contain '1' we shall chose only open subsets of X containing '1'

$$1 \in \{1\} \subset \{1,3\}.$$
$$x \in G \subset N$$

nbd. of 3 are $\{1,3,4\}, \{1,2,3,4\}, \{1,3,4,5\}, X$

nbd. of 5 are $\{1,2,5\}, \{1,2,3,5\}, \{1,2,4,5\}, X$